AS Further Mathematics Unit 1: Further Pure Mathematics A
General instructions for marking GCE Mathematics

1. The mark scheme should be applied precisely and no departure made from it. Marks should be awarded directly as indicated and no further subdivision made.

2. Marking Abbreviations
The following may be used in marking schemes or in the marking of scripts to indicate reasons for the marks awarded.

cao = correct answer only
MR = misread
PA = premature approximation
bod = benefit of doubt
oe = or equivalent
si = seen or implied
ISW = ignore subsequent working

F.T. = follow through (indicates correct working following an error and indicates a further error has been made)

Anything given in brackets in the marking scheme is expected but, not required, to gain credit.

3. Premature Approximation
A candidate who approximates prematurely and then proceeds correctly to a final answer loses 1 mark as directed by the Principal Examiner.

4. Misreads
When the data of a question is misread in such a way as not to alter the aim or difficulty of a question, follow through the working and allot marks for the candidates' answers as on the scheme using the new data.
This is only applicable if a wrong value, is used consistently throughout a solution; if the correct value appears anywhere, the solution is not classed as MR (but may, of course, still earn other marks).

5. Marking codes

- 'M' marks are awarded for any correct method applied to appropriate working, even though a numerical error may be involved. Once earned they cannot be lost.
- 'm' marks are dependant method marks. They are only given if the relevant previous 'M' mark has been earned.
- 'A' marks are given for a numerically correct stage, for a correct result or for an answer lying within a specified range. They are only given if the relevant M/m mark has been earned either explicitly or by inference from the correct answer.
- 'B' marks are independent of method and are usually awarded for an accurate result or statement.
- 'S' marks are awarded for strategy
- 'E' marks are awarded for explanation
- 'U' marks are awarded for units
- 'P' marks are awarded for plotting points
- 'C' marks are awarded for drawing curves

AS Further Mathematics Unit 1: Further Pure Mathematics A

Solutions and Mark Scheme

Qu. No.	Solution	Mark	AO	Notes
1.	When n = 1, $4^n + 2 = 6$ which is divisible by 6 so the proposition is true for n = 1.	B1	AO2	
	Assume the proposition to be true for $n = k$ so that $4^k + 2$ is divisible by 6 and equals $6N$	M1	AO2	
	Consider (for $n = k + 1$)	M1	AO2	
	$4^{k+1} + 2 = 4 \times 4^k + 2$	A1	AO2	
	$= 4 \times (6N - 2) + 2$	A1	AO2	
	$= 24N - 6$	A1	AO2	
	This is divisible by 6 so true for $k \Rightarrow$ true for $k + 1$. Since true for $n = 1$, the result is proved by induction.	A1	AO2	
		[7]		
2.(a)	$\dfrac{-1+7\text{i}}{2+\text{i}} = \dfrac{(-1+7\text{i})(2-\text{i})}{(2+\text{i})(2-\text{i})}$	M1	AO3	
	$= \dfrac{5+15\text{i}}{5}$	A1	AO3	
	$= 1 + 3\text{i}$	A1	AO3	
	$2(x + \text{i}y) + \text{i}(x - \text{i}y) = 1 + 3\text{i}$	M1	AO3	FT from line above
	$2x + y = 1$	A1	AO3	
	$x + 2y = 3$	A1	AO3	
	$\left(x = -\dfrac{1}{3};\ y = \dfrac{5}{3}\right)$			
	$z = \dfrac{-1+5\text{i}}{3}$	A1	AO1	
(b)	$\lvert z \rvert = \dfrac{\sqrt{26}}{3} = 1.70$ (1.699673171)	B1	AO1	
	$\tan^{-1}(-5) = -1.3734...$	B1	AO1	
	$\arg(z) = -1.3734 + \pi = 1.77$ (1.768191887)	B1	AO1	
	$z = 1.70(\cos 1.77 + \text{i}\sin 1.77)$	B1	AO1	
		[11]		

Qu. No.	Solution	Mark	AO	Notes
3.	$S_n = \sum_{r=1}^{n} r(r+1)(r+3)$	M1	AO1	
	$= \sum_{r=1}^{n} (r^3 + 4r^2 + 3r)$	A1	AO1	
	$= \frac{n^2(n+1)^2}{4} + 4\frac{n(n+1)(2n+1)}{6} + 3\frac{n(n+1)}{2}$	A1	AO1	
	$= \frac{n(n+1)}{12}\left(3n(n+1) + 8(2n+1) + 18\right)$	A1	AO1	
	$= \frac{n(n+1)}{12}\left(3n^2 + 19n + 26\right)$	A1	AO1	
	$= \frac{n(n+1)(n+2)(3n+13)}{12}$	A1 **[6]**	AO1	
4.(a)	$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\beta\gamma + \gamma\alpha + \alpha\beta)$	M1	AO2	
	$= 4^2 - 2\times 14$ $= -12$	A1	AO2	
	A cubic equation either has 3 real roots or 1 real root. Since the sum of squares is negative, the 3 roots cannot all be real so there is just 1 real root	B1	AO2	
(b)	A second root is $1 - 3\text{i}$,	B1	AO1	
	since complex roots occur in conjugate pairs.	E1	AO2	
	The third root is 2	B1	AO1	
	since the sum of the 2 complex roots is 2 and the sum of the 3 roots is 4,	E1	AO2	
		[7]		

Qu. No.	Solution	Mark	AO	Notes
5.	Putting $z = x + \mathrm{i}y$,	M1	AO3	
	$\lvert x - 3 + \mathrm{i}y\rvert = 2\lvert x + \mathrm{i}(y + 1)\rvert$	A1	AO3	
	$(x-3)^2 + y^2 = 4\left(x^2 + (y+1)^2\right)$	m1	AO3	
	$x^2 - 6x + 9 + y^2 = 4x^2 + 4y^2 + 8y + 4$			
	$3x^2 + 3y^2 + 6x + 8y - 5 = 0$	A1	AO3	
	This is the equation of a circle			
	$x^2 + y^2 + 2x + \frac{8}{3}y = \frac{5}{3}$	M1	AO1	
	$(x+1)^2 + \left(y + \frac{4}{3}\right)^2 = \frac{5}{3} + 1 + \frac{16}{9}$	m1	AO1	
	Centre = $\left(-1, -\frac{4}{3}\right)$	A1	AO1	
	Radius = $\sqrt{\frac{5}{3} + 1 + \frac{16}{9}}$	M1	AO1	
	$= 2.11 \quad \left(\frac{2\sqrt{10}}{3}\right)$	A1	AO1	
		[9]		

Qu. No.	Solution	Mark	AO	Notes
6.(a)	Reflection matrix = $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	B1	AO1	
	Translation matrix = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$	B1	AO1	
	Rotation matrix = $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	B1	AO1	
	$\mathbf{T} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	M1	AO1	
	$= \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	A1	AO1	
	$= \begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	A1	AO1	
(b)	Fixed points satisfy $\begin{bmatrix} -1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$	M1	AO1	
	$-x+2=x; y+1=y$	A1	AO1	
	The second equation is inconsistent so there are no fixed points.	A1	AO2	
		[9]		

Qu. No.	Solution	Mark	AO	Notes
7.(a)	Putting $z = x + \mathrm{i}y$ and $w = u + \mathrm{i}v$, $u + \mathrm{i}v = (x + \mathrm{i}y)(x + 1 + \mathrm{i}y)$ v = imaginary part = $y(x + 1) + xy$ $= y(1 + 2x)$ u = real part = $x(x + 1) - y^2$	M1 A1 A1 A1	AO1 AO1 AO1 AO1	
(b)	$u = x(x+1) - (x+1)^2$ $= -x - 1$ $v = (x+1)(2x+1)$ $= -u(-2u-1)$ $= 2u^2 + u$	M1 A1 M1 A1 A1 **[9]**	AO3 AO3 AO3 AO3 AO3	
8.(a)(i)	$\mathbf{AB} = (2\mathbf{i}+3\mathbf{j}+\mathbf{k})-(\mathbf{i}+2\mathbf{j}+3\mathbf{k})$ $= \mathbf{i}+\mathbf{j}-2\mathbf{k}$	M1 A1	AO1 AO1	
(ii)	Equation of line is $\mathrm{r} = \mathbf{i}+2\mathbf{j}+3\mathbf{k}+\lambda(\mathbf{i}+\mathbf{j}-2\mathbf{k})$	B1	AO1	
(b)(i)	The line cuts the plane where $1 + \lambda + 3(2 + \lambda) - 2(3 - 2\lambda) = 5$ $\lambda = \frac{1}{2}$ Point of intersection = $\left(\frac{3}{2}, \frac{5}{2}, 2\right)$	 M1 A1 A1	 AO1 AO1 AO1	
(ii)	Direction of normal = $(1, 3, -2)$ If θ denotes the angle between L and the normal, $\cos\theta = \frac{(1,3,-2).(1,1,-2)}{\|(1,3,-2)\|\|(1,1,-2)\|}$ $= \frac{8}{\sqrt{14}\sqrt{6}}$ $\theta = 29.2(0593\ldots)^{\circ}$ Angle between L and plane = 60.8°.	B1 M1 A1 A1 A1 A1 **[12]**	AO1 AO1 AO1 AO1 AO1 AO1	